Problems and Applications

1. a. To solve for the steady-state value of y as a function of s, n, g, and δ, we begin with the equation for the change in the capital stock in the steady state:

$$\Delta k = sf(k) - (\delta + n + g)k = 0.$$

The production function $y = \sqrt{k}$ can also be rewritten as $y^2 = k$. Plugging this production function into the equation for the change in the capital stock, we find that in the steady state:

$$sy - (\delta + n + g)y^2 = 0.$$

Solving this, we find the steady-state value of y:

$$y^* = s/(\delta + n + g).$$

b. The question provides us with the following information about each country:

Developed country:	$s = 0.28$	Less-developed country:	$s = 0.10$
	$n = 0.01$		$n = 0.04$
	$g = 0.02$		$g = 0.02$
	$\delta = 0.04$		$\delta = 0.04$

Using the equation for y^* that we derived in part (a), we can calculate the steady-state values of y for each country.

Developed country: $y^* = 0.28/(0.04 + 0.01 + 0.02) = 4.$

Less-developed country: $y^* = 0.10/(0.04 + 0.04 + 0.02) = 1.$

c. The equation for y^* that we derived in part (a) shows that the less-developed country could raise its level of income by reducing its population growth rate n or by increasing its saving rate s. Policies that reduce population growth include introducing methods of birth control and implementing disincentives for having children. Policies that increase the saving rate include increasing public saving by reducing the budget deficit and introducing private saving incentives such as I.R.A.'s and other tax concessions that increase the return to saving.

2. To solve this problem, it is useful to establish what we know about the U.S. economy:

A Cobb–Douglas production function has the form $y = k^{\alpha}$, where α is capital's share of income. The question tells us that $\alpha = 0.3$, so we know that the production function is $y = k^{0.3}$.

In the steady state, we know that the growth rate of output equals 3 percent, so we know that $(n + g) = 0.03$.

The depreciation rate $\delta = 0.04$.

The capital–output ratio $K/Y = 2.5$. Because $k/y = [K/(L \times E)]/[Y/(L \times E)] = K/Y$, we also know that $k/y = 2.5$. (That is, the capital–output ratio is the same in terms of effective workers as it is in levels.)

a. Begin with the steady-state condition, $sy = (\delta + n + g)k$. Rewriting this equation leads to a formula for saving in the steady state:

$$s = (\delta + n + g)(k/y).$$

Plugging in the values established above:

$$s = (0.04 + 0.03)(2.5) = 0.175.$$

The initial saving rate is 17.5 percent.

b. We know from Chapter 3 that with a Cobb–Douglas production function, capital's share of income $\alpha = MPK(K/Y)$. Rewriting, we have:

$$MPK = \alpha/(K/Y).$$

5. How do differences in education across countries affect the Solow model? Education is one factor affecting the *efficiency of labor*, which we denoted by E. (Other factors affecting the efficiency of labor include levels of health, skill, and knowledge.) Since country 1 has a more highly educated labor force than country 2, each worker in country 1 is more efficient. That is, $E_1 > E_2$. We will assume that both countries are in steady state.
   a. In the Solow growth model, the rate of growth of total income is equal to $n + g$, which is independent of the work force's *level* of education. The two countries will, thus, have the same rate of growth of total income because they have the same rate of population growth and the same rate of technological progress.
   b. Because both countries have the same saving rate, the same population growth rate, and the same rate of technological progress, we know that the two countries will converge to the same steady-state level of capital per efficiency unit of labor k^*. This is shown in Figure 8–1.

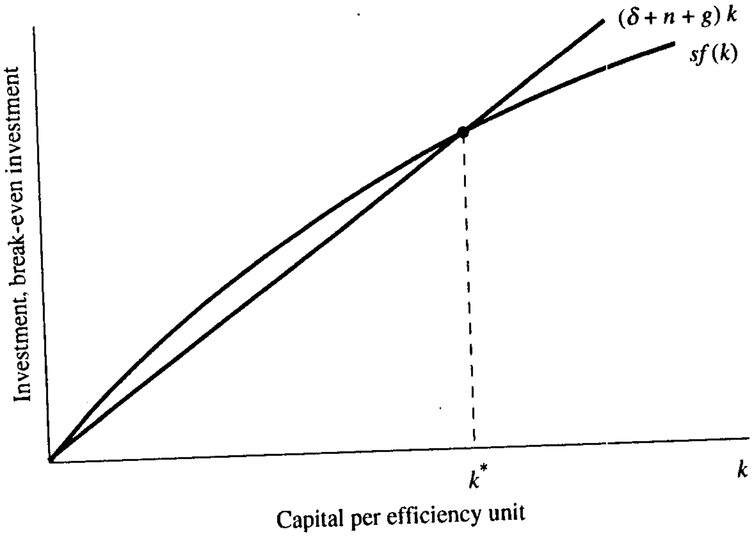

Figure 8–1

   Hence, output per efficiency unit of labor in the steady state, which is $y^* = f(k^*)$, is the same in both countries. But $y^* = Y/(L \times E)$ or $Y/L = y^* E$. We know that y^* will be the same in both countries, but that $E_1 > E_2$. Therefore, $y^*E_1 > y^*E_2$. This implies that $(Y/L)_1 > (Y/L)_2$. Thus, the level of income per worker will be higher in the country with the more educated labor force.
   c. We know that the real rental price of capital R equals the marginal product of capital (MPK). But the MPK depends on the capital stock per efficiency unit of labor. In the steady state, both countries have $k_1^* = k_2^* = k^*$ because both countries have the same saving rate, the same population growth rate, and the same rate of technological progress. Therefore, it must be true that $R_1 = R_2 = MPK$. Thus, the real rental price of capital is identical in both countries.
   d. Output is divided between capital income and labor income. Therefore, the wage per efficiency unit of labor can be expressed as:

$$w = f(k) - MPK \cdot k.$$

   As discussed in parts (b) and (c), both countries have the same steady-state capital stock k and the same MPK. Therefore, the wage per efficiency unit in the two countries is equal.

   Workers, however, care about the wage per unit of labor, not the wage per efficiency unit. Also, we can observe the wage per unit of labor but not the wage per efficiency unit. The wage per unit of labor is related to the wage per efficiency unit of labor by the equation

$$\text{Wage per Unit of } L = wE.$$