Homework for week 5

Suppose you evaluate your today's consumption and tomorrow's consumption as $u(c_0, c_1) = c_0 * c_1^\beta$ where $\beta = 0.99$. You have a fixed income of $(Y_0, Y_1) = (10, 10)$ and the market interest rate is given as $r = 0.05$

1. What is the demand for current and future consumption? Show the derivation process as well as the numerical answer

2. Suppose the market interest rate rises to $r = 0.10$. How did the demand change? Explain both numerically and in words

(Answer)
(part1) Define intertemporal budget constraint as

$$\begin{aligned} C_2 &\leq Y_2 + (1+r)(Y_1 - C_1) \\ (1+r)C_1 + C_2 &\leq Y_2 + (1+r)Y_1 \\ C_1 + \frac{1}{1+r}C_2 &\leq Y_1 + \frac{1}{1+r}Y_2 \equiv Y^{PV} \end{aligned}$$

The tangency condition states

$$MRS = -(1+r)$$

from our functional form this can be rewritten as

$$\begin{aligned} -\frac{C_2}{\beta C_1} &= -(1+r) \\ C_2 &= \beta(1+r)C_1 \end{aligned}$$

plugging into budget constraint yields

$$\begin{aligned} C_1 + \frac{\beta(1+r)C_1}{1+r} &\leq Y^{PV} \\ C_1 &= \frac{1}{1+\beta}Y^{PV} \end{aligned}$$

and

$$C_2 = \frac{\beta(1+r)}{1+\beta}Y^{PV}$$

given our parameter values we have

$$\begin{aligned} C_1 &= \frac{1}{1+\beta}Y^{PV} \\ &= 19.5/1.99 \\ &= 9.8 \end{aligned}$$

plugging into the tangency condition gives

$$\begin{aligned} C_2 &= \beta(1+r)C_1 \\ &= 0.99 * 1.05 * 9.8 \\ &= 10.2 \end{aligned}$$

unlike the numerical example in the class, this consumer starts as a **saver**.

(part2) if interest rises, consumption becomes as follows

$$\begin{aligned} C_1 &= \frac{1}{1+\beta} Y^{PV} \\ &= 19.1/1.99 \\ &= 9.6 \end{aligned}$$

and

$$\begin{aligned} C_2 &= \beta(1+r)C_1 \\ &= 0.99 * 1.1 * 9.6 \\ &= 10.4 \end{aligned}$$

Current consumption decreased because present value of lifetime income (Y^{PV}) has decreased. Future consumption increased because increase in saving has increased income in period 2.